14th lecture

ELECTROWEAK INTERACTIONS

– part 2 –

- $\mathcal{L} \sim i\bar{\psi}\gamma^\mu D_\mu \psi$ structure, but...

  what is D_μ for any particular $\psi \in \{e, \nu_e, u, d, \dots\}$?

  know: interaction with $W^\pm_\mu$, Z_μ should be L/R-asymmetric

- Weinberg & Salam in 1967/68 found right solution:
  - left-handed fermions couple to $\hat{W}_\mu$ & B_μ

    generators $t_a = \frac{1}{2}\sigma_a$ ← SU(2) doublet   carry $U_Y(1)$ charge Y_L
  - right-handed fermions couple to $\hat{W}_\mu$ & B_μ

    they do <u>not</u> couple! ⟺ SU(2) singlet   carry other weak hypercharge Y_R

    ↳ generators $t_a = 0$

different numbers for different particles

- look at 1^{st} generation ($\nu = \nu_e$): $\begin{pmatrix}\nu\\ e\end{pmatrix}_L$, ν_R, e_R; $\begin{pmatrix}u\\ d\end{pmatrix}_L$, u_R, d_R

$$\left.\begin{aligned} \rightsquigarrow D_\mu \begin{pmatrix}\nu\\ e\end{pmatrix}_L &= \left(\partial_\mu + g\hat{W}_\mu + \tfrac{i}{2} g' Y_L B_\mu\right)\begin{pmatrix}\nu\\ e\end{pmatrix}_L \\ D_\mu e_R &= \left(\partial_\mu + \tfrac{i}{2} g' Y_R B_\mu\right) e_R \end{aligned}\right\} \Rightarrow \text{table of values } \{Y_L, Y_R\}$$

where those weak hypercharges are

ψ	ν_L, e_L	ν_R	e_R	u_L, d_L	u_R	d_R
Y	-1	0	-2	$1/3$	$4/3$	$-2/3$

$\phi = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix}$

$Y = 1$

note: $Y = 2\times$ average electric charge in an $SU(2)$ multiplet

e.g. $Y_{u_L} = Y_{d_L} = 2\times \frac{1}{2}\left[2/3 + (-1/3)\right] = 1/3$ ✓

turn around: given $\{Y\} \longrightarrow$ compute $\{q\}$ (inside $SU(2)$ doublet always $\Delta q = 1$)

the actual values for $\{Y\}$ determined by anomaly cancellation!

↳ exercise ☺

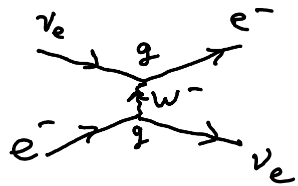
$$q = \tfrac{1}{2} Y + T_3$$ ← weak isospin ($\frac{1}{2}\sigma_3$)

- Full 1^{st} generation (electroweak) fermion Lagrangian:

$$\mathcal{L}_f = i(\bar\nu_L\ \bar e_L)\gamma^\mu\left(\partial_\mu + g\hat W_\mu - \frac{i}{2}g' B_\mu\right)\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$$
$$+ i\bar\nu_R\gamma^\mu\partial_\mu\nu_R + i\bar e_R\gamma^\mu(\partial_\mu - ig'B_\mu)e_R$$
$$+ i(\bar u_L\ \bar d_L)\gamma^\mu\left(\partial_\mu + g\hat W_\mu + \frac{i}{6}g' B_\mu\right)\begin{pmatrix}u_L\\ d_L\end{pmatrix}$$
$$+ i\bar u_R\gamma^\mu\left(\partial_\mu + \frac{2i}{3}g'B_\mu\right)u_R + i\bar d_R\gamma^\mu\left(\partial_\mu - \frac{i}{3}g'B_\mu\right)d_R$$

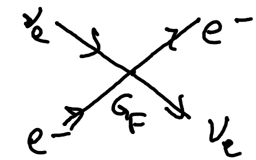
$$\hat W_\mu = -\frac{i}{2}\sigma_a W^a_\mu = -\frac{i}{2}\begin{bmatrix}W^3 & \sqrt2 W^- \\ \sqrt2 W^+ & -W^3\end{bmatrix}_\mu$$
$$B_\mu = \mathbb{1}B_\mu = \begin{bmatrix}B & 0\\ 0 & B\end{bmatrix}_\mu$$

- ν_R : interacts with nothing (only gravitationally) $\rightsquigarrow$ "sterile"
  not completely true: Yukawa interaction + Higgs effect $\rightsquigarrow$ ν mass
  $\rightarrow$ coupling $\bar\nu_L\nu_R + \bar\nu_R\nu_L$ converts ν_R to ν_L $\rightsquigarrow$ interacts weakly
- $\bar f f W^\pm$: $\frac{g}{\sqrt2}W^-_\mu(\bar\nu_L\gamma^\mu e_L + \bar u_L\gamma^\mu d_L) + \text{c.c.}$



$$\rightsquigarrow \mathcal{M}^{SM}_{\nu e\to\nu e} = \left(\frac{g}{\sqrt2}\right)^2\cdot \bar u_{e_L}\gamma^\mu u_{\nu_L}\frac{\eta_{\mu\nu}}{q^2 - M_W^2}\bar u_{\nu_L}\gamma^\nu u_{e_L}$$

$\downarrow$



$$\mathcal{M}^{\text{Fermi}}_{\nu e\to\nu e} = \frac{G_F}{\sqrt2}\cdot 2\bar u_{e_L}\gamma^\mu u_{\nu_L}\cdot 2\bar u_{\nu_L}\gamma_\mu u_{e_L}$$

comparison $\rightsquigarrow$ $G_F = g^2/4\sqrt2 M_W^2$ $\rightsquigarrow$ need to know g !

- $\bar{f}fA$: $g\sin\theta_W\, A_\mu\left(\bar{e}\gamma^\mu e - \frac{2}{3}\bar{u}\gamma^\mu u + \frac{1}{3}\bar{d}\gamma^\mu d\right)$

with $\bar{e}\gamma^\mu e = \bar{e}_L\gamma^\mu e_L + \bar{e}_R\gamma^\mu e_R$ etc., L/R symmetric

read off electric charge: $e = -g\sin\theta_W = -\frac{gg'}{\sqrt{g^2+g'^2}}$

$$\rightsquigarrow M_W^2 = \frac{g^2}{4\sqrt{2}G_F} = \frac{e^2/G_F}{4\sqrt{2}\sin^2\theta_W} \overset{e^2=4\pi\alpha}{=} \frac{\pi\alpha}{\sqrt{2}G_F\cdot\sin^2\theta_W} \approx \left(\frac{37.3}{\sin\theta_W}\,\text{GeV}\right)^2$$

- $\bar{f}fZ$: $\frac{1}{2}Z_\mu\sqrt{g^2+g'^2}\Big(\bar{\nu}_L\gamma^\mu\nu_L - \cos 2\theta_W\,\bar{e}_L\gamma^\mu e_L + 2\sin^2\theta_W\,\bar{e}_R\gamma^\mu e_R$

$$+\left(1-\frac{4}{3}\sin^2\theta_W\right)\bar{u}_L\gamma^\mu u_L - \frac{4}{3}\sin^2\theta_W\,\bar{u}_R\gamma^\mu u_R$$

$$+\left(\frac{2}{3}\sin^2\theta - 1\right)\bar{d}_L\gamma^\mu d_L + \frac{2}{3}\sin^2\theta_W\,\bar{d}_R\gamma^\mu d_R\Big)$$

$\rightsquigarrow$ interaction of Z with weak neutral current $j^\mu_{(\nu)} + \dots$

$$\rightsquigarrow M_Z^2 = M_W^2/\cos^2\theta_W = \frac{\pi\alpha\cdot 4}{\sqrt{2}G_F\cdot\sin^2 2\theta_W} \approx \left(\frac{74.6}{\sin 2\theta_W}\,\text{GeV}\right)^2$$

- these tree-level results get slightly modified by loop corrections

experimentally: $M_W \approx 80.4$ GeV, $M_Z \approx 91.2$ GeV

$\rightsquigarrow \sin^2\theta_W \approx 0.23$ (at $E \approx M_W$)

roughly: $\theta_W \approx 30°$ $\rightsquigarrow$ $\sin\theta_W \approx 1/2$, $\sin^2\theta_W \approx 1/4$, $\sin 2\theta_W \approx \cos\theta_W \approx \frac{1}{2}\sqrt{3}$

Standard Model fermions: masses

experimentally, fermions have masses $\rightsquigarrow m(\bar\Psi_L \Psi_R + \bar\Psi_R \Psi_L)$

but Ψ_L are $SU(2)$ doublets while Ψ_R are singlets $\rightsquigarrow$

$\rightsquigarrow$ no $SU(2)$-invariant mass term possible!

- Higgs mechanism comes to the rescue:

  add Yukawa interactions $\phi^\dagger \bar\Psi_R \Psi_L \sim (\cdot\ \cdot)\, \bar\Psi_R \begin{pmatrix}\cdot\\ \cdot\end{pmatrix}$ $SU(2)$ & $U_Y(1)$ inv.

  Higgs doublet $\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix} = \begin{pmatrix}\phi^+\\ \phi^0\end{pmatrix} \xrightarrow{\text{VEV}} \begin{pmatrix}0\\ v\end{pmatrix}$, coupling $h_\cdot$ for e, ν, u, d

$$-\mathcal{L}_e^{Yuk} = -h_e(\phi^- \ \bar\phi^0)\, \bar e_R \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \text{c.c.} \xrightarrow{\text{VEV}} -h_e (0\ v)\, \bar e_R \begin{pmatrix}\nu_L\\ e_L\end{pmatrix} + \text{c.c.} = -h_e v \bar e e$$

$$-\mathcal{L}_d^{Yuk} = -h_d(\phi^- \ \phi^0)\, \bar d_R \begin{pmatrix}u_L\\ d_L\end{pmatrix} + \text{c.c.} \xrightarrow{\text{VEV}} -h_d (0\ v)\, \bar d_R \begin{pmatrix}u_L\\ d_L\end{pmatrix} + \text{c.c.} = -h_d v \bar d d$$

for ν & u use antifundamental doublets $\begin{pmatrix}\bar e_L\\ -\bar\nu_L\end{pmatrix}$ & $\begin{pmatrix}\bar d_L\\ -\bar u_L\end{pmatrix}$ equiv. to $\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}$ & $\begin{pmatrix}u_L\\ d_L\end{pmatrix}$

$$-\mathcal{L}_\nu^{Yuk} = +h_\nu(\phi^- \ \bar\phi^0) \begin{pmatrix}\bar e_L\\ -\bar\nu_L\end{pmatrix} \nu_R + \text{c.c.} \xrightarrow{\text{VEV}} h_\nu (0\ v) \begin{pmatrix}\bar e_L\\ -\bar\nu_L\end{pmatrix} \nu_R + \text{c.c.} = -h_\nu v \bar\nu \nu$$

$$-\mathcal{L}_u^{Yuk} = +h_u(\phi^- \ \bar\phi^0) \begin{pmatrix}\bar d_L\\ -\bar u_L\end{pmatrix} u_R + \text{c.c.} \xrightarrow{\text{VEV}} h_u (0\ v) \begin{pmatrix}\bar d_L\\ -\bar u_L\end{pmatrix} u_R + \text{c.c.} = -h_u v \bar u u$$

$\rightsquigarrow$ masses $m_f = h_f \cdot v$ traced back to Yukawa couplings

generations and their mixing

full fermion-gauge interactions includes generation mixing

- $\bar{f}fW^\pm$: $W^-_\mu j^{+\mu} + W^+_\mu j^{-\mu}$ with

$$j^+_\mu = \bar{\nu}'_{eL}\gamma_\mu e_L + \bar{\nu}'_{\mu L}\gamma_\mu \mu_L + \bar{\nu}'_{\tau L}\gamma_\mu \tau_L$$
$$+ \bar{u}'_L\gamma_\mu d_L + \bar{c}'_L\gamma_\mu s_L + \bar{t}'_L\gamma_\mu b_L$$

where primed $\begin{Bmatrix}\text{leptons}\\ \text{quarks}\end{Bmatrix}$ are related to unprimed ones via $\begin{Bmatrix}\text{PMNS}\\ \text{CKM}\end{Bmatrix}$ matrix

- can we not just redefine $\psi' \to \psi$? ok if massless !
  but Yukawa interactions contain ψ and not ψ' !
  → gauge & Yukawa interactions diagonal in different bases
  ⇝ after VEV: gauge eigenstates $\xleftrightarrow{\text{rotation}}$ mass eigenstates
- why rotate only the left-handed fields ?
  flavor-changing currents/interactions affect only left-handed fields
  right-handed fields may also be flavor-rotated, but
  $\bar{f}fA$ & $\bar{f}fZ$ is invariant under this → irrelevant

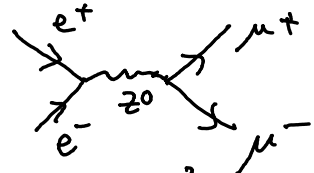
back to the zoo

- electroweak breaking scale ~ 100 GeV
  above it, weak interactions as important as e.m. ones!

- example $e^+e^- \longrightarrow \mu^+\mu^-$

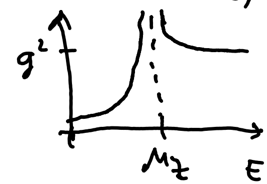
$\mathcal{M}_{e^+e^- \to \mu^+\mu^-} =$

$+$



$E =$ CMS energy

$\sim g^2$ $\qquad \sim g^2 \dfrac{E^2}{|M_Z^2 - E^2|}$



new effects:

- asymmetry: μ^- prefers to go in e^+ direction
- polarization: final $\mu^+\mu^-$ are polarized even if e^+e^- were not

- W & Z production & decay (SPS CERN 1983)

$p\bar{p} \longrightarrow W^\pm + \text{hadrons}$, $\quad W \to \ell\nu$ or 2 quark jets

$p\bar{p} \longrightarrow Z + \text{hadrons}$, $\quad Z \to \ell\bar{\ell}$ or — " —

cleaner at LEP: $e^+e^- \longrightarrow Z \longrightarrow$ anything

beautiful resonances: $\Gamma_W \approx 2.1$ GeV, $\Gamma_Z \approx 2.5$ GeV, but Γ/M small

interesting: $Z \to \nu\bar{\nu}$ contribute with $G_F M_Z^3 / 12\sqrt{2}\pi \approx 170$ MeV to $\Gamma_Z \rightsquigarrow$ only 3 generations!

· the Higgs $\quad m_H \approx 125$ GeV $\quad$ (LHC CERN 2012)

H(x) describes the fluctuations of $|\phi(x)|$ around its VEV v

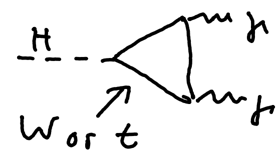
H couples to fermions via Yukawa interaction with strength $h_f = m_f/v$

→ strongest coupling to heavy quarks $(t, b, \ldots)$!

decays: $H \to t\bar{t}$ kinematically forbidden

$H \to b\bar{b}$ → 2 quark jets (~ half of decays)

$H \to \gamma\gamma$ much cleaner but 1-loop:



$H \to \underset{\text{virtual}}{ZZ} \to (e^+e^-, \mu^+\mu^-, \ldots)^2$

→ peak in distribution of invariant masses

the detection was an effort of thousands of scientists !

neutrino oscillations

- solar neutrinos: created in principal process

$$pp \to d\, e^+ \nu_e + 0.42 \text{ MeV} \qquad (d = \text{deuterium})$$

↳ flies into space & may hit Earth

cross section for ν_e capture at 1 MeV is $\sigma \sim 10^{-44}\,\text{cm}^2$

mean free path in matter of particle density n is

$$\lambda \sim (n\sigma)^{-1} \approx 100 \text{ light years}$$

collisions can be observed → solar neutrino flux involved

- Homestake experiment (1970–94, Raymond Davis)

$\sim$ 350.000 ℓ of $Cl_2C=CCl_2$ $\sim$ 1500 m underground

solar neutrinos induce $\nu_e + {}^{37}Cl \to {}^{37}Ar + e^-$

result: ν_e flux was 2–3 times lower than solar model predicts

- resolution: solar ν_e on their way to Earth oscillate into ν_μ & ν_τ

for simplicity consider just two generations, mixing with θ_p

- Yukawa or mass eigenstates ν_e, ν_μ, denote $|\nu_1\rangle$ & $|\nu_2\rangle$ with masses m_1 & m_2
- gauge interaction eigenstates ν_e', ν_μ', denote $|\nu_e\rangle$ & $|\nu_\mu\rangle$ created in the sun

mixture: $$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta_p & \sin\theta_p \\ -\sin\theta_p & \cos\theta_p \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

- $t=0$: creation of solar neutrinos

$$|\nu_e\rangle(0) = \cos\theta_p\,|\nu_1\rangle(0) + \sin\theta_p\,|\nu_2\rangle(0)$$

$t>0$: time evolution as plane waves:

$$|\nu_i\rangle(\vec{r},t) = e^{i(\vec{p}_i\cdot\vec{r} - E_i t)}\,|\nu_i\rangle(0) \qquad i=1,2$$

$m_i \ll E \rightsquigarrow$ ultrarelativistic $\rightsquigarrow$

$$E = \sqrt{\vec{p}^2 + m^2} = |\vec{p}|\sqrt{1 + m^2/\vec{p}^2} \approx |\vec{p}|\left(1 + \frac{m^2}{2\vec{p}^2}\right) \overset{\vec{p}^2 \approx E^2}{\approx} |\vec{p}| + \frac{m^2}{2E}$$

put $\vec{r} = (L,0,0)$ & $t \approx L \rightsquigarrow$ phase $\approx pL - EL = -\frac{m^2 L}{2E}$ differs for $i=1,2$

$\rightsquigarrow$ at distance L superposition changed to

$$\sim e^{-i m_1^2 L/2E} \cos\theta_p\,|\nu_1\rangle + e^{-i m_2^2 L/2E} \sin\theta_p\,|\nu_2\rangle$$

$\rightsquigarrow$ nonzero projection $\langle\nu_\mu|\nu_e(L)\rangle \sim \left(e^{-i m_2^2 L/2E} - e^{-i m_1^2 L/2E}\right)\cos\theta_p \sin\theta_p$

⇝ muon neutrinos appear along the way!

"Rabi oscillations"

- $P_{\nu_e \to \nu_e}(L) = \left| \langle \cos\theta_p \, \nu_1(0) + \sin\theta_p \, \nu_2(0) \,|\, \cos\theta_p \, \nu_1(L) + \sin\theta_p \, \nu_2(L) \rangle \right|^2$

$$= \left| \cos^2\theta_p \, e^{-i\frac{m_1^2 L}{2E}} + \sin^2\theta_p \, e^{-i\frac{m_2^2 L}{2E}} \right|^2$$

$\Delta m^2 = m_2^2 - m_1^2$

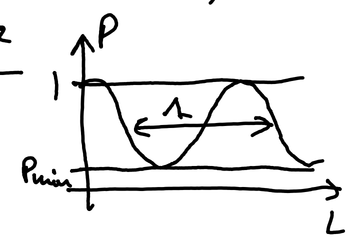
$$= \cos^4\theta_p + \sin^4\theta_p + \sin^2\theta_p \cos^2\theta_p \left(e^{iL\Delta m^2/2E} + e^{-iL\Delta m^2/2E} \right)$$

$$= \underbrace{(\cos^2\theta_p + \sin^2\theta_p)^2}_{} + 2\sin^2\theta_p \cos^2\theta_p \left(-1 + \cos\frac{L\Delta m^2}{2E} \right)$$

$1 - \cos x = 2\sin^2\frac{x}{2}$

$2\sin x \cos x = \sin 2x$

$$= 1 - \sin^2 2\theta_p \cdot \sin^2 \frac{L\Delta m^2}{4E}$$



with $P_{min} = \cos^2 2\theta_p$ & $\Lambda = \frac{4\pi E}{\Delta m^2}$

experimentally: $\theta_p \approx 45°$ ⇝ $P_{min} \approx 0$, $\Delta m^2 \sim 10^{-4}\,\mathrm{eV}^2$, $E \sim 1\,\mathrm{MeV}$ ⇝ $\Lambda = \mathcal{O}(\text{kilometers})$

→ on average, only ½ of the ν are ν_e and e^-
[ν_μ has not enough energy to create μ^-]

- historically, people doubted this result for long time but Davis & Bahcall did not relent $\longrightarrow$ 2002 Nobel

- credibility only after confirmation by other experiments
  - reactor & accelerator experiment detectors at distance of $\sim 10\,m$ to $1000\,km$
  - Super-Kamiokande: cosmic rays $\rightarrow$ atmospheric ν_μ these convert to ν_τ inside Earth

  Nobel 2015

- today we know:

  $\Delta m^2_{21} \approx 7.5 \times 10^{-5}\,eV^2$, $\Delta m^2_{31} \approx \Delta m^2_{32} \approx 2.5 \times 10^{-3}\,eV^2$

  masses themselves not accessible

  but astrophysics: $m_1 + m_2 + m_3 \lesssim 0.12\,eV$

Wrap - Up

- SM is quite complex but ...

... it very accwately describes a humongous amount of physical phenomena:

all of non-gravitational physiss!

- but do you like it?

probably not $\rightarrow$ too complex to be beautiful

particle degrees of freedom per generation = 16

another measure: # input parameters, count:

— Λ_{QCD} & θ_{QCD} (multiplying $\varepsilon^{\mu\nu\rho\lambda} \mathrm{tr}\, \hat{G}_{\mu\nu} \hat{G}_{\rho\lambda}$, is $\lesssim 10^{-10}$)

— g, g', v, λ

— Yukawa h_f for all 3×4 fermions

— mixing angles in CKM & PMNS
4+4

26 parameteD

- also some logical inconsistencies:
  - Higgs mass gets 1-loop correction  $\sim \lambda \int^{\Lambda} \frac{d^4p}{p^2-m_H^2} \sim \lambda \cdot \Lambda^2$

    renormalizability: cancel by counterterm $\sim \Lambda^2 \phi^* \phi$

    $\rightsquigarrow$ needs extreme fine-tuning! "naturalness problem"
  - coupling constants g', λ, h_f in Abelian & Higgs sector grow with energy

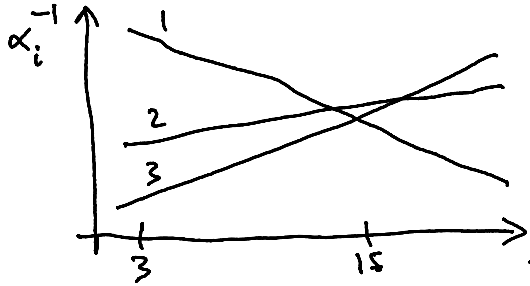
    $\rightsquigarrow$ need a nonperturbative completion at high energies "triviality"

- remedies
  - supersymmetry (SUSY):

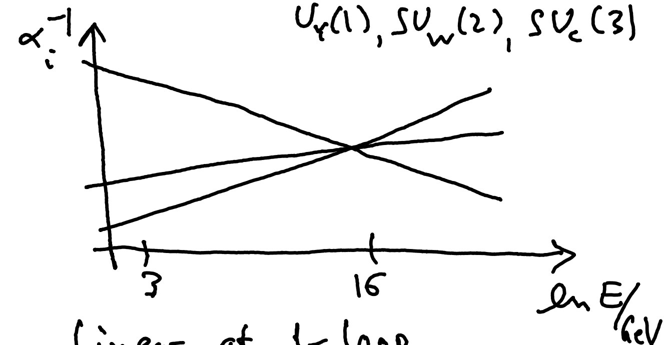
    technically solves naturalness problem ($m_H^2 \sim \ln \Lambda$ only) but must be broken at $\gtrsim 1$ TeV

— Grand Unification (GUT)

$i = 1,2,3$ for $U_Y(1), SU_W(2), SU_c(3)$



$\Longrightarrow$



where $\alpha_i^{-1} = \frac{4\pi}{g_i^2} \sim 1 + b_i \ln E/E_0$ linear at 1-loop

minimal GUT : $SU(3) \times SU(2) \times U(1) \subset SU(5)$ $\rightsquigarrow$ 24 gauge bosons

(Georgi–Glashow 1974)

particle content: $\underline{\bar{5}} = \begin{bmatrix} d_r \\ d_g \\ d_b \\ e^- \\ \nu \end{bmatrix}$ & $\underline{10} = (5 \times 5)_{\text{antisym.}}$

predicts charge quantization and $\sin^2\theta_W = 3/8$ at E_{GUT}

$$\begin{pmatrix} \hat{G}_\mu & \hat{X}_\mu \\ \hat{X}_\mu^T & \hat{W}_\mu \end{pmatrix}$$

$\rightsquigarrow$ q–l transitions

SU(5) broken at $\sim 10^{15}$ GeV $\rightsquigarrow$ $m_X \sim 10^{15}$ GeV

but $\tau_p \approx 10^{28}$ y, however $\tau_p^{\exp} \gtrsim 10^{34}$ y ↯

— SUSY GUTs : compatible with experiment but <u>many</u> new parameters ☹

<u>missing:</u>
dark matter
dark energy
qu. gravity
black holes
Big Bang